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One-Particle Excitation Spectrum in a Bisoliton Model of Superconductivity

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A bisoliton model of high-temperature superconductivity is formulated in terms of one-particle states. The transition to a superconducting state is shown to be accompanied by the energy spectrum rearrangement. The dielectric gap typical for BCS theory is appeared. Its dependence on concentration of carriers is of nonmonotonous character. The conditions for realization of a bisoliton mechanism of superconductivity are considered.

Сформулирована бисолитонная модель высокотемпературной сверхпроводимости в терминах одночастичных состояний. Показано, что переход в сверхпроводящее состояние сопровождается перестройкой энергетического спектра. Появляется диэлектрическая щель, характерная для теории БКШ. Ее зависимость от концентрации носителей носит немонотонный характер. Рассматриваются условия реализации бисолитонного механизма сверхпроводимости.

1. Introduction

The properties of optical and tunneling phenomena in superconducting systems are generated, primarily, by the peculiarities inherent to the one-particle energy spectrum of carriers. The transition to a superconducting state is accompanied by the appearance of a dielectric gap and changes in the dispersion law of quasi-particle states. For low-temperature superconductors these peculiarities are described mainly by the BCS theory [1]. However, for high-temperature superconductors, the problem of a correct description of the one-particle excitation spectrum is not yet solved since there is no clear concept of the superconductivity mechanism.

In our paper we consider the peculiarities of changes in a one-particle spectrum under the transition into a superconducting state in a bisoliton model of high-temperature superconductivity. According to this theory [2, 3] Fermi particles in a quasi-one-dimensional chain in states with energy close to the Fermi level with opposite wave numbers and spins are paired due to the deformation energy producing a bisoliton with zero spin and doubled charge. Bisolitons distributed periodically can move along the chain with the same velocity forming a stable bisoliton condensate [4].

Below we formulate a bisoliton model of superconductivity in terms of one-particle states. In this case we are interested in the temperature range close to zero.

2. The Deformation Field

The transition to a superconducting state with arising bisoliton condensate is accompanied by the generation of a local deformation field. In a bisoliton model this field is described by the potential [2]

$$V(\xi) = -4Jg\Phi^2(\xi)\,, \tag{2.1}$$

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where $\xi = (x - vt)/a$, a is the period of the chain, v the velocity of bisoliton motion. The constants J and g are determined, respectively, by

$$J = \frac{\hbar^2}{2ma^2}\,.$$

$$g = \frac{\sigma^2}{2J\varkappa(1 - s^2)}\,; \qquad s = \frac{v}{v_0}\,.$$

Here g is the deformation interaction parameter, $\varkappa$ the elasticity coefficient, v_0 the sound velocity in the chain, m is the Fermi-particle mass.

The function $\Phi(\xi)$ is periodic with period aL (L^{-1} has the meaning of a bisoliton concentration) and is normalized by the condition

$$\int\limits_0^L \Phi^2(\xi)\,\mathrm{d}\xi = 1\,. \tag{2.2}$$

It can be represented by the Jacobi elliptic functions dn (u, q)

$$\Phi(\xi) = \left(\frac{g}{2}\right)^{1/2} \frac{1}{\mathrm{E}(q)}\,\mathrm{dn}\,(u, q)\,; \qquad u = \frac{g\xi}{\mathrm{E}(q)}\,, \tag{2.3}$$

where the modulus q is determined by

$$gL = 2\mathrm{E}(q)\,\mathrm{K}(q)\,. \tag{2.4}$$

$\mathrm{K}(q)$ and $\mathrm{E}(q)$ are total elliptic integrals of first and second kinds, respectively, [4].

Thus, according to (2.1), the deformation field represents a system of potential wells distributed periodically, with period aL. Each potential well involves two quasi-particles with opposite spins. This field will rearrange the spectrum of excitations of one-particle states.

3. One-Particle States

The wave function $\Psi(\xi)$ of one-particle stationary states in the coordinate system moving with the bisoliton condensate, is a solution to the Schrödinger equation with potential $V(\xi)$ determined by (2.1),

$$\left(J\frac{\mathrm{d}^2}{\mathrm{d}\xi^2} + V(\xi) + E + E_\mathrm{F} - W\right)\Psi(\xi) = 0\,. \tag{3.1}$$

Here W is the chain deformation energy of a single particle,

$$W = 2Jg\int\limits_0^L \Phi^4(\xi)\,\mathrm{d}\xi\,,$$

$E_\mathrm{F} = Jk_\mathrm{F}^2a^2$ is the Fermi energy.

To determine the one-particle spectrum we represent $\Psi(\xi)$ as the expansion in functions $\varphi_k(\xi)$,

$$\varphi_k(\xi) = \sqrt{\frac{1}{l}}\,\exp\,\{iak\xi\}\,; \qquad l \to \infty\,, \tag{3.2}$$

orthonormalized in the chain of length la

$$\Psi(\xi) = \sum_k u(k)\,\varphi_k(\xi)\,. \tag{3.3}$$

The function $\Psi(\xi)$ is normalized in the space l and, respectively,

$$\sum_k |u(k)|^2 = 1 .$$

Using (3.3) we represent (3.1) as

$$(-\mathscr{E}(k) + E + E_F - W)\, u(k) + \sum_{k_1} V(k - k_1)\, u(k_1) = 0 \tag{3.4}$$

in which

$$\mathscr{E}(k) = Ja^2k^2 ,$$

$$V(k - k_1) = \int_{-l/2}^{l/2} \varphi_k^*(\xi)\, V(\xi)\, \varphi_{k_1}(\xi)\, d\xi . \tag{3.5}$$

Further simplification of (3.4) can be realized using the properties of the potential $V(\xi)$

$$V(k - k_1) = U(k - k_1) \frac{L}{l} \sum_{n=-l/2L}^{l/2L} \exp \{-iLan(k - k_1)\} , \tag{3.6}$$

where

$$U(k) = \frac{1}{L} \int_{-L/2}^{L/2} V(\xi) \exp \{-ika\xi\}\, d\xi . \tag{3.7}$$

$V(k - k_1)$ is nonzero, if

$$L(k - k_1)\, a = 2\pi\nu ; \qquad \nu = 0, +1, +2 . \tag{3.8}$$

The density of quasi-particles being in the states satisfying (3.8) should follow L^{-1}. This requirement is fulfilled for particles in the states for which

$$(k - k_1) = 2k_F ; \qquad k_1 = k - 2k_F . \tag{3.9}$$

Denoting $u(k - 2k_F) = v(k)$ and $U(2k_F) = \Delta$, $U(0) = 2Jg\ 1/L$, the system of equations (3.4) will take the form

$$\begin{cases} (-\mathscr{E}(k) + E + \mu)\, u(k) + \Delta v(k) = 0 , \\ \Delta u(k) + \big(-\mathscr{E}(k - 2k_F) + E + \mu\big)\, v(k) = 0 , \end{cases} \tag{3.10}$$

$$u^2(k) + v^2(k) = 1 ; \qquad \mu = E_F - U(0) + W .$$

The one-particle excitation spectrum follows from (3.3):

$$E(k) = \frac{\mathscr{E}(k) + \mathscr{E}(k - 2k_F) - 2\mu}{2} \pm \sqrt{\frac{[\mathscr{E}(k) - \mathscr{E}(k - 2k_F)]^2}{4} + \Delta^2} , \tag{3.11}$$

and also the values $u^2(k)$ and $v^2(k)$ as the function of wave number k

$$\begin{aligned} u^2(k) &= \frac{1}{2}\left\{1 - \frac{\mathscr{E}(k) - \mathscr{E}(k - 2k_F)}{\sqrt{[\mathscr{E}(k) - \mathscr{E}(k - 2k_F)]^2 + 4\Delta^2}}\right\}, \\ v^2(k) &= \frac{1}{2}\left\{1 + \frac{\mathscr{E}(k) - \mathscr{E}(k - 2k_F)}{\sqrt{[\mathscr{E}(k) - \mathscr{E}(k - 2k_F)]^2 + 4\Delta^2}}\right\}. \end{aligned} \tag{3.12}$$

Thus, the dielectric gap of width 2Δ is formed in a one-particle excitation spectrum. The sign "plus" in (3.11) corresponds to the upper subband and "minus" to the lower one. Fig. 1 illustrates the energy structure of a one-particle spectrum in the superconducting state. Actually, such a picture of the spectrum is connected with

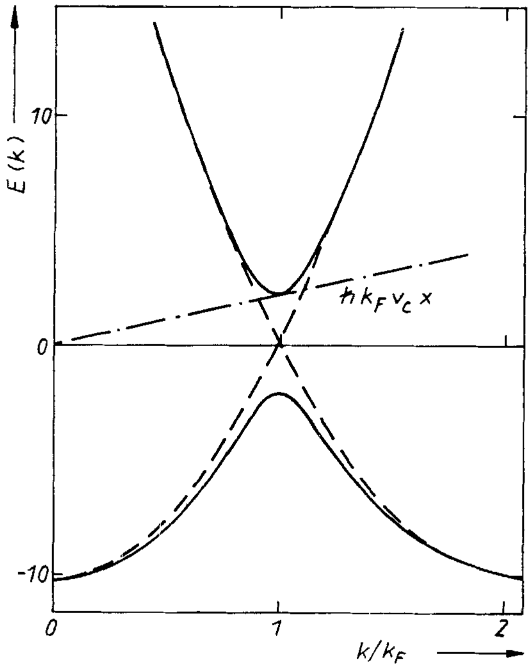

Fig. 1. Energy structure of the one-particle spectrum in the superconducting state (solid curves) and in normal state (dash-dotted line). $\mu = 10$, $\Delta = 2$ (in rel. units)

hybridization of states with energies close to the Fermi level but being distinguished by $2k_F$.

For energies close to the Fermi level $|\mathcal{E}(k) - \mu| \sim \Delta \ll E_F$ there holds

$$\mathcal{E}(k) - E_F \approx E_F - \mathcal{E}(k - 2k_F) .$$

Taking the latter into account (3.11) and (3.12) take the form

$$E(k) = \pm \sqrt{[\mathcal{E}(k) - E_F]^2 + \Delta^2}\, W + U(0) , \tag{3.13}$$

$$u^2(k) = \frac{1}{2}\left\{1 - \frac{\mathcal{E}(k) - E_F}{\sqrt{[\mathcal{E}(k) - E_F]^2 + \Delta^2}}\right\},$$
$$v^2(k) = \frac{1}{2}\left\{1 + \frac{\mathcal{E}(k) - E_F}{\sqrt{[\mathcal{E}(k) - E_F]^2 + \Delta^2}}\right\}. \tag{3.14}$$

The wave function of one-particle states $\Psi(\xi)$ will have the form for the lower band

$$\Psi_H(\xi) = \sqrt{\frac{1}{l}} \exp\{ika\xi\} [u(k) + v(k) \exp\{-2ika\xi\}] \tag{3.15}$$

and for the upper one

$$\Psi_b(\xi) = \sqrt{\frac{1}{l}} \exp\{ika\xi\} [v(k) - u(k) \exp\{-2ik_F a\xi\}] . \tag{3.16}$$

The lower band states are occupied, those of the upper band are empty.

4. The Dielectric Gap

The dielectric gap value is determined by $U(k)$ for $k = 2k_F$. Using (2.2), (2.3), and (3.7), having calculated the relevant integrals, we get

$$\Delta = \frac{8\pi k_F a J}{E(q)\, L} \operatorname{sh}^{-1} \left[\frac{2k_F a E(q)\, K'(q)}{g}\right], \tag{4.1}$$

where $K'(q) = K(\sqrt{1 - q^2})$.

At small bisoliton concentrations $L \to \infty$ (which corresponds to $q \to 1$) (4.1) takes the form

$$\Delta = \frac{8\pi k_F J a}{L} \operatorname{sh}^{-1} \left(\frac{k_F \pi a}{g}\right). \tag{4.2}$$

Using the state density value $N(E)$ in a one-dimensional system near the Fermi energy $E = E_F$,

$$N(E_F) = \frac{2}{l} \sum_k \delta(\mathcal{E}(k) - E_F) = \frac{1}{2\pi k_F a J}, \tag{4.3}$$

the expression for the gap Δ can be represented as

$$\Delta = 2\hbar\omega \operatorname{sh}^{-1} \left\{-\frac{1}{\lambda}\right\}. \tag{4.4}$$

Here

$$\lambda = 2JgN(E_F) = GN(E_F)\,,$$

$$G = 2Jg\,, \qquad \hbar\omega = \frac{4\pi k_F a J}{L}.$$

The parameter λ has the same meaning as in the relations for the dielectric gap value in the BCS theory. It characterizes the dimensionless electron–phonon interaction constant. $\hbar\omega$ depends on the bisoliton concentration and the following relation holds:

$$\hbar\omega N(E_F) = \frac{2}{L}\,; \qquad \hbar\omega \ll E_F\,. \tag{4.5}$$

The latter arises from condition (3.9).

Thus, $\hbar\omega$ corresponds to the interval of energy values at which Fermi particles participate in creating the superconducting condensate. If ν is constant, Δ as function of k_F as shown in Fig. 2, has its maximum value in the region k_F satisfying the condi-

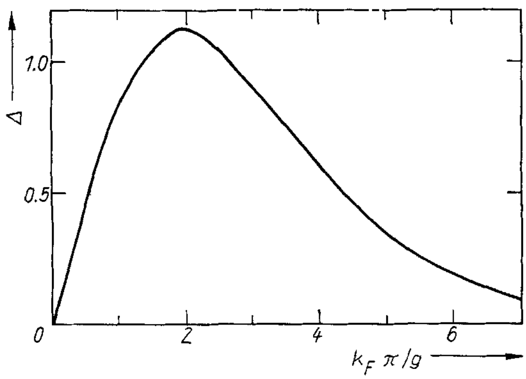

Fig. 2. Dependence of Δ on wave number $\frac{\pi k_F}{g}$ $(= \lambda^{-1} = x)$

tion

$$ak_F \approx \frac{2g}{\pi} \quad \text{or} \quad \lambda = 0.5\,.$$

Matching the conditions (2.4) and (4.5) in the region of large bisoliton concentrations, we get the value $\nu = 1$ at which Δ is maximum. In the one-dimensional case this is consistent with the bisoliton concentration k_F/π which testifies the fact that all particles are involved in generating a superconducting condensate. In the general case the parameter ν^{-1} determines the relative number of particles responsible for superconductivity. Its value is determined, under condition (3.8), by the deformation energy value W strongly dependent on the structure and mechanical properties of the superconductor.

Under condition (4.5) for $\hbar\omega$ we have

$$\hbar\omega_m = \frac{2^5}{\pi^3}\frac{g^2 J}{\nu}\,. \tag{4.6}$$

The value $\hbar\omega_m$ is the analogue of the Debye energy $\hbar\omega_D$ in BCS theory.

5. Conclusion

Thus, analogous to the BCS theory, in a bisoliton theory the one-particle excitation spectrum is rearranged when going over to the superconducting state. The dependence of the dielectric constant on the parameter λ is typical for the BCS theory, but the pre-exponential factor has a different value determined by (4.4) and (4.6). Comparing ω_D and ω_m we find from realizing the bisoliton superconductivity mechanism,

$$G > G_{cr} = \frac{\pi}{2}\sqrt{\hbar\omega_D J \pi \nu}\,.$$

The above condition can be realized at small values ν. A specific value ν follows from the possibility to realize deformation in a superconducting system. With $G < G_{cr}$ the BCS superconductivity mechanism will produce higher values for the critical temperatures. Since $\hbar\omega_m$ is ν dependent, with availability of microstructures typical for ceramic superconductors, the existence of Δ with different values ν is possible.

In [5] microcontact spectroscopy of the superconductor $La_{1.8}Sr_{0.2}CuO_4$ revealed two gaps having the values $2\Delta_1 = 13.3$ meV and $2\Delta_2 = 26$ meV. This, according to the above-presented model of superconductivity, corresponds to $\nu = 1$, $\nu = 2$, and $\Delta_2/\Delta_1 = 2$. It is essential that in this case the current–voltage dependences prove to be typical for the excitation spectrum of type (3.13).

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